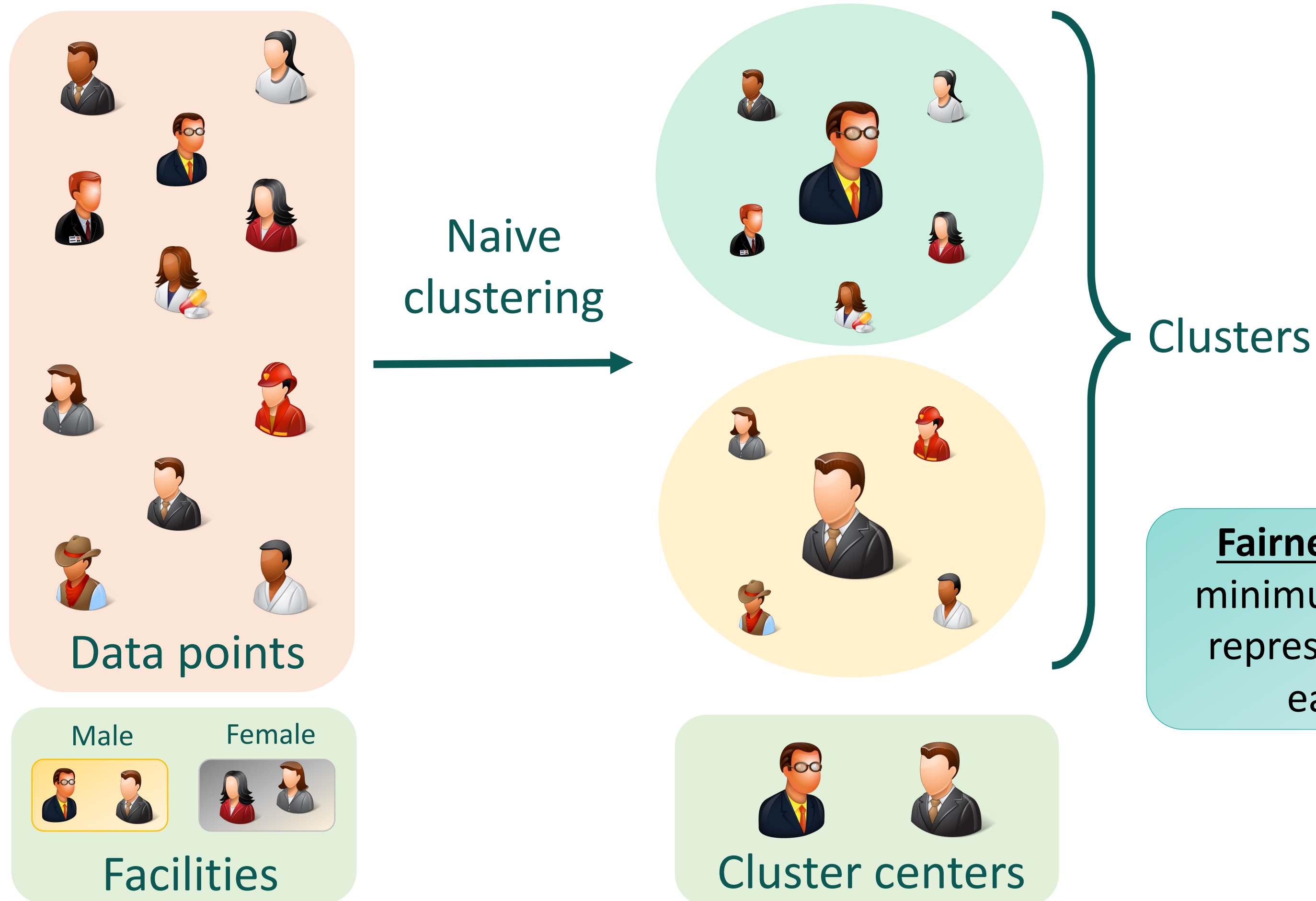
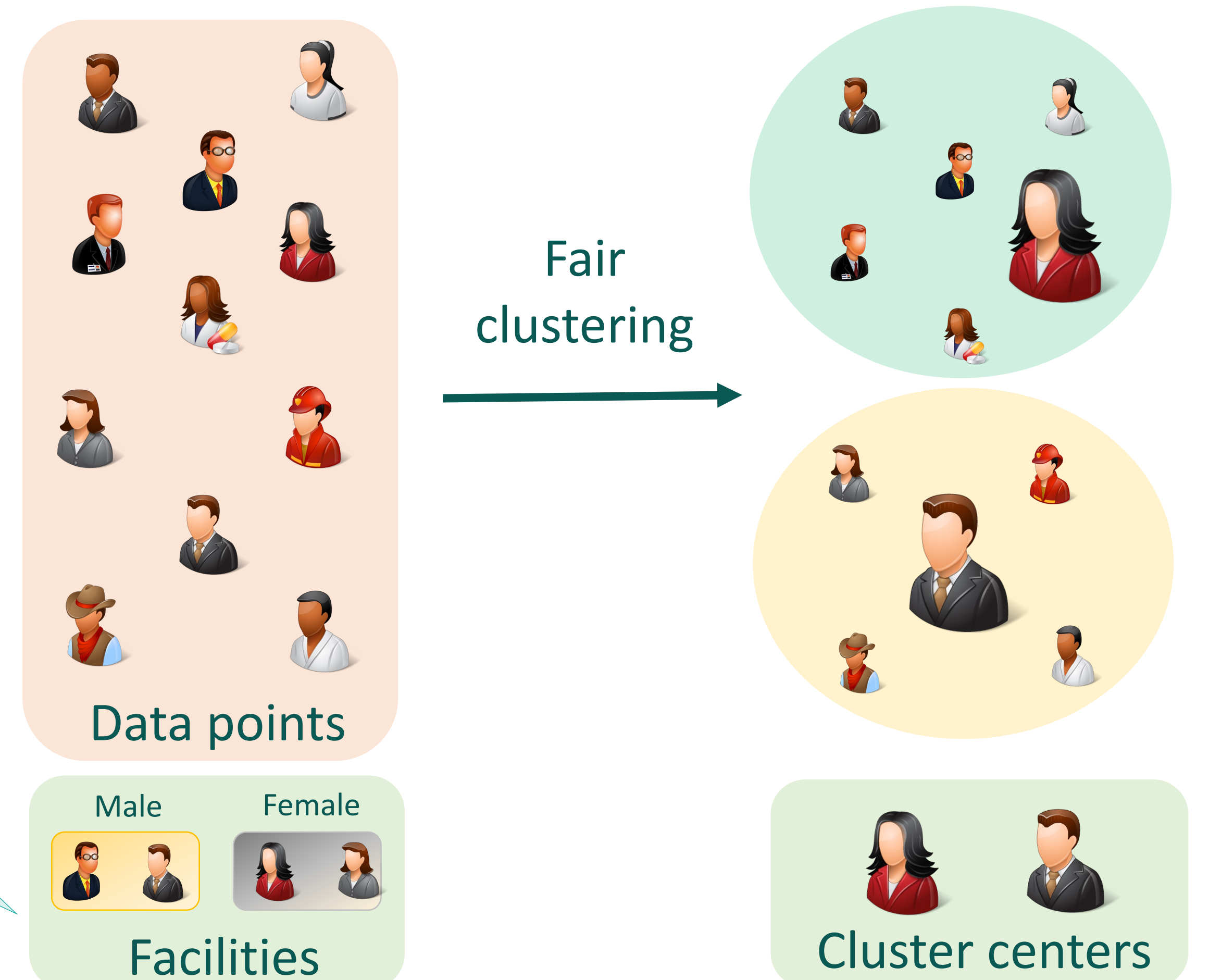


## The $k$ -supplier problem



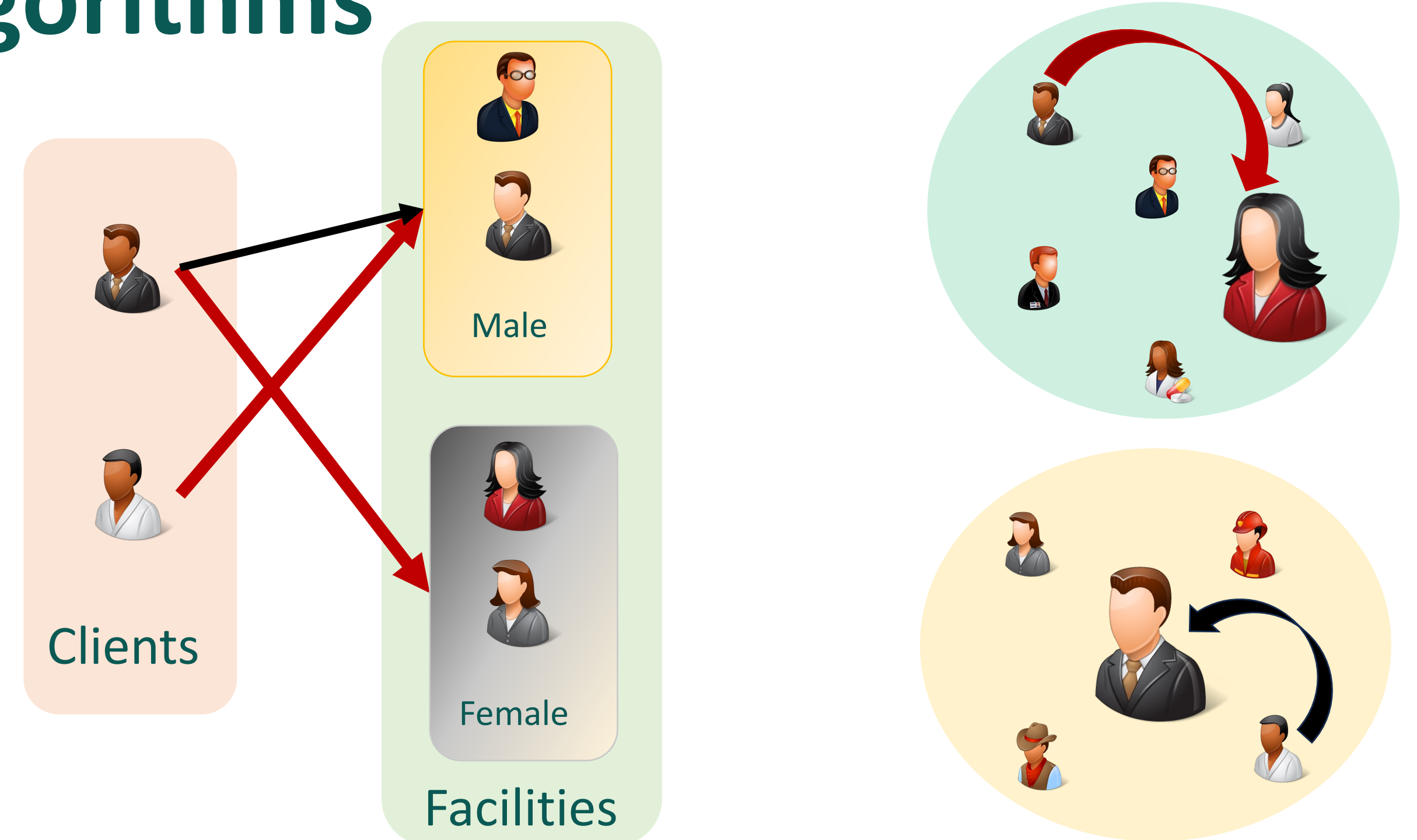
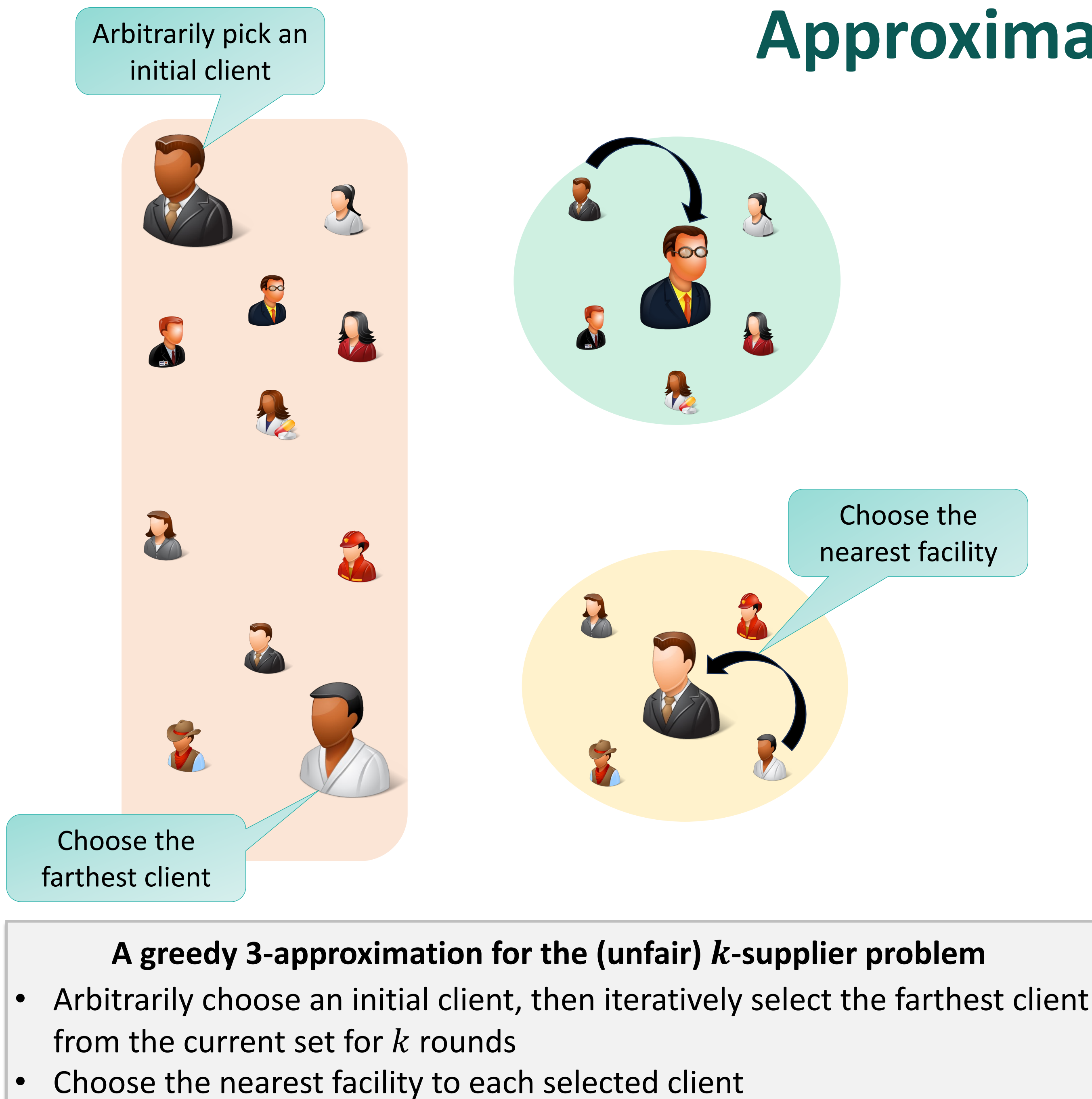
Cluster the data points by choosing  $k = 2$  representative centers / facilities to minimize the maximum distance from any point to its closest representative

## The fair $k$ -supplier problem



The goal remains to minimize the maximum distance, but with fairness constraints: each group must contribute a minimum (and a maximum) number of cluster centers

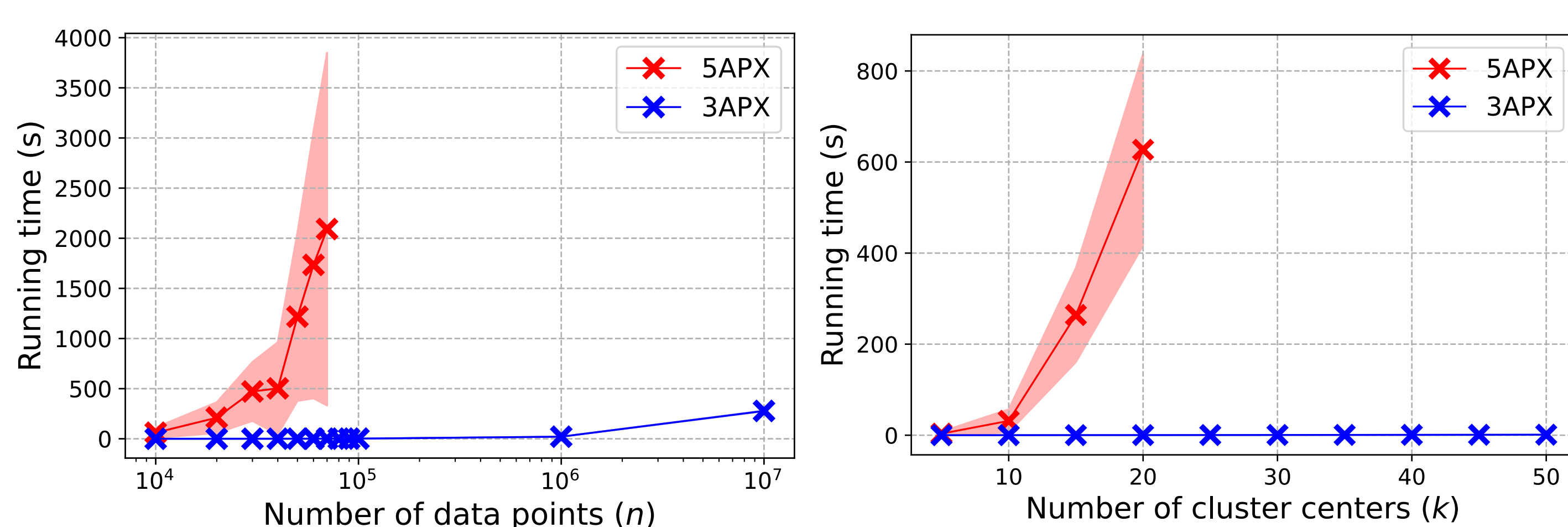
## Approximation algorithms



### A 3-approximation for the fair $k$ -supplier problem using matching framework

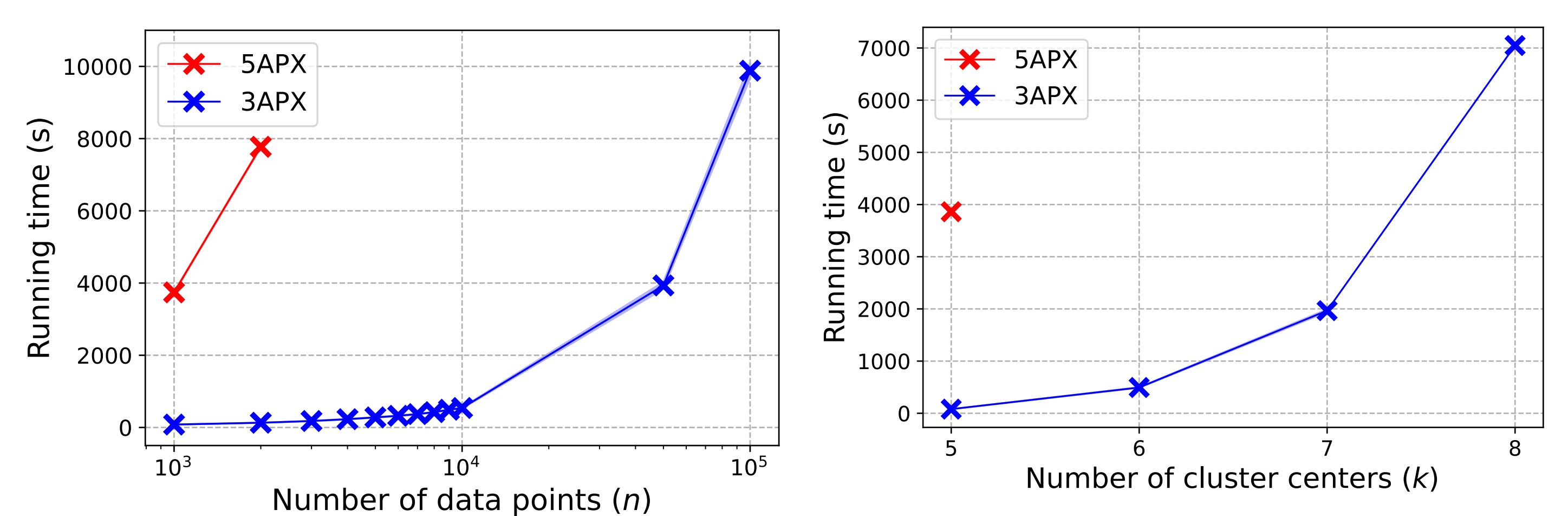
- Arbitrarily choose an initial client, then iteratively select the farthest client from the current set for  $k$  rounds
- Guess the optimal radius using binary search (over at most  $nk$  values)
- Build a bipartite graph: connect a client to each group within the guessed radius
- Find a maximum matching and select one facility per matched group within the radius
- This yields a 3-approximation in  $O((kn + k^2\sqrt{k})\log n \log k)$  time
- For intersecting groups, reduce the problem to multiple instances with disjoint groups, incurring an overhead of  $O(2^{tk})$
- $n$  is number of data points and  $t$  is number of groups

## Experimental results



### Salability of our 3-approximation algorithm (3-APX) vs. Chen et al.'s (TCS, 2024) 5-approximation (5-APX) for fair $k$ -supplier with disjoint groups

- Left plot:** Runtime scaling with increasing number of data points  $n$ , for  $k = 10$  and 5 disjoint groups
- Right plot:** Runtime scaling with increasing number of cluster centres  $k$ , for a fixed dataset of  $n = 10\,000$  points
- Fairness constraint:** select an equal number of cluster centres from each group



### Salability of our 3-approximation algorithm (3-APX) vs. Thejaswi et al.'s (ArXiv, 2024) 5-approximation (5-APX) for fair $k$ -supplier with intersecting groups

- Left plot:** Runtime scaling with increasing number of data points  $n$ , for  $k = 5$  and 5 intersecting groups. Fairness constraint: choose at least 2 cluster centres from each group
- Right plot:** Runtime scaling with increasing number of cluster centres  $k$ , for a fixed dataset of  $n = 1000$  points. Fairness constraint: choose  $\frac{2k}{t}$  cluster centres from each group